Sind folgende Funktionen auf $\mathbb{R}$ diff.bar? 8.2

(a)

$f: \mathbb{R} \to \mathbb{R}$

$f(x) = 2x^4 - 5x$

Def.: $f: D \subseteq \mathbb{R} \to \mathbb{R}$ heißt diff.bar an d Stelle $A \in D$ falls

$f'(A) = \lim_{h \to 0} \frac{f(A+h) - f(A)}{h}$ (= Grenzwert) existiert.

" Diff.tial. quot.

h-Methode

$$f'(x) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} =$$

$$= \lim_{h \to 0} \frac{2\cdot(x_0+h)^4 - 5\cdot(x_0+h) - (2\cdot x_0^4 - 5\cdot x_0)}{h} =$$

$$= \lim_{h \to 0} \frac{2\cdot(x_0^4 + 4\cdot x_0^3\cdot h + 6\cdot x_0^2 h^2 + 4\cdot x_0 h^3 + h^4) \cancel{-5x_0} - 5h - 2x_0^4 \cancel{+ 5\cdot x_0}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x_0^4} + 8x_0^3 h + 12x_0^2 h^2 + 8x_0 h^3 + 2h^4 - 5h \cancel{- 2x_0^4}}{h}$$

$$= \lim_{h \to 0} 8x_0^3 - 5 + (12x_0^2 h + 8x_0 h^2 + 2h^3) =$$

$$\underline{\underline{8x_0^3 - 5}}$$

$$f'(x) = 8x^3 - 5$$

(b) $f: \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} x^2, & x \leq 0 \\ -x^2, & x > 0 \end{cases} \qquad f'(x) = \begin{cases} 2x, & x \leq 0 \\ -2x, & x > 0 \end{cases}$$

$$f(x) = x^2$$

$$f'(x) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} = \quad \cancel{f(x+}$$

$$\lim_{h \to 0} \frac{(x_0+h)^2 - x_0^2}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{x_0^2} + 2x_0h + h^2 \cancel{- x_0^2}}{h} =$$

$$\lim_{h \to 0} 2x_0 + h = 2 \cdot x_0$$

$$f(x) = -x^2$$

$$f'(x) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} =$$

$$\lim_{h \to 0} \frac{-(x_0+h)^2 - (-x^2)}{h} =$$

$$\lim_{h \to 0} \frac{-(x_0^2 + 2x_0h + h^2) + x^2}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{-x_0^2} - 2x_0h - h^2 \cancel{+x^2}}{h} =$$

$$\lim_{h \to 0} -2x_0 - h = -2x_0$$

(b) Fortsetzung

$$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \frac{f(h)}{h} = \dots$$

Fall 1 $\quad h < 0 \quad$ (linkss. GW)

$$\dots \lim_{h \to 0} \frac{h^2}{h} = \lim_{h \to 0} h = \underline{\underline{0}}$$

Fall 2 $\quad h > 0 \quad$ (rechtss. GW)

$$\dots \lim_{h \to 0} \frac{-h^2}{h} = \lim_{h \to 0} -h = \underline{\underline{0}}$$

$\Rightarrow$ $f(x)$ diff.bar auf $\mathbb{R}$

(da bei $f'(0)$ ls. GW = rs. GW)

& für $f'(x)$ für $x \neq 0$ $\exists f'(x)$

c) $f: \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} 2x & x \leq 0 \\ -x & x > 0 \end{cases}$$

$$f'(x) = \begin{cases} 2 & x \leq 0 \\ -1 & x > 0 \end{cases}$$

$f(x) = 2x$

$$f'(x) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} = \lim_{h \to 0} \frac{2 \cdot (x_0+h) - 2 \cdot x_0}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x_0} + 2\cancel{h} - \cancel{2x_0}}{\cancel{h}} = 2$$

$f(x) = -x$

$$f'(x) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} = \lim_{h \to 0} \frac{-(x_0+h) - (-x_0)}{h}$$

$$= \lim_{h \to 0} \frac{-x_0 - h + x_0}{h} = -1$$

$$f'(0) = \lim_{h \to 0} \frac{f(h)}{h} = \dots \qquad \frac{f(\overset{0}{x_0}+h) - f(\overset{0}{x_0})}{h}$$

Fall 1: $h < 0$ (ls. Gw)

$$\dots = \lim_{h \to 0} \frac{2h}{h} = 2$$

$\Longrightarrow$ $f(x)$ nicht auf $\mathbb{R}$ diffbar (nur auf $\mathbb{R} \setminus \{0\}$)

Fall 2: $h > 0$ (rs. Gw)

$$\dots = \lim_{h \to 0} \frac{-h}{h} = -1$$